

Math 55 Quiz 1 DIS 105

Name: _____

31 Jan 2022

1. Let $C(x, y)$ be the statement “Person x has been to country y .” Express each of these sentences in terms of $C(x, y)$, quantifiers, and logical connectives, where the domain for x consists of everyone in the world and for y consists of all countries.

(a) John has been to France, Germany, and Italy. [2 points]

(b) Everyone has been to at least two different countries. [2 points]

(c) There exists someone who has been to every country in the world except for the USA. [3 points]

(a) $C(\text{John}, \text{France}) \wedge C(\text{John}, \text{Germany}) \wedge C(\text{John}, \text{Italy})$

(b) $\forall x \exists y_1 \exists y_2 C(x, y_1) \wedge C(x, y_2) \wedge (y_1 \neq y_2)$

(c) $\exists x (\forall y (C(x, y) \vee (y = \text{the USA}))) \wedge \neg C(x, \text{the USA})$

2. Prove that if x is an odd number, then $x^2 + x + 1$ is an odd number. [3 points]

Suppose x is an odd number. Then there exists an integer n such that $x = 2n + 1$. Then $x^2 + x + 1 = (2n + 1)^2 + (2n + 1) + 1 = 4n^2 + 6n + 3 = 2(2n^2 + 3n + 1) + 1$, where $2n^2 + 3n + 1$ is an odd number. Hence $x^2 + x + 1$ is an odd number.